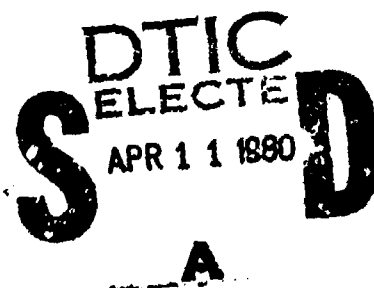CM/CCM-79-4

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Adaptive Antenna Systems

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U.S. Army Materiel Development
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The basic principles of adaptive antenna systems are reviewed. Special attention is given to the sidelobe canceller--the prototype and classic example of adaptive antenna systems. The operation of the sidelobe canceller is interpreted in terms of noise cancelling and adaptive null steering. The classical theory of adaptive elements is developed. Tapped-delay-line and frequency-domain array-processing filters are discussed. The signal-to-noise ratio and the mean-square-error performance criteria are distinguished. The general theory of adaptive noise

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cancelling is linked to the sidelobe canceller, the classical theory of adaptive elements, and the adaptive notch filter. An adaptive antenna system is derived from the constrained minimum power criterion, which limits the cancelation of the desired signal.

CONTENTS

FIGURES

1. INTRODUCTION

One who seeks to familiarize himself with adaptive antenna systems for interference rejection is confronted by a vast literature containing many different proposed systems. However, there are only a few fundamentally different concepts underlying the variations. In this paper, these concepts and their relationships are stressed. Adaptive systems that are not intended for interference rejection, such as retrodirective arrays, are not considered.

Special attention is given to the sidelobe canceller, the archetypical adaptive antenna system. The initial mathematical analysis is heuristic in order to clarify the basic principles of operation. It is shown that the adaptation in a sidelobe canceller can be interpreted not only as noise cancellation, but also as adaptive beam forming and null steering. The analysis of the sidelobe canceller leads naturally to consideration of the multiple sidelobe canceller. A slight modification of the sidelobe canceller yields a notch filter.

After the initial heuristic analysis of the sidelobe canceller, the remainder of this paper contains a more rigorous mathematical analysis. Alternative adaptive systems that can handle wide-bandwidth signals are developed from the classical theory. There are two forms of the adaptive elements. In one form, the weight-adjustment mechanism responds to the system output; in the other form, the mechanism responds to the difference between the output and the desired response. The inputs to the adaptive elements may be derived from tapped-delay-line or frequency-domain array-processing filters.

The detailed design of an adaptive antenna system is dependent upon the performance criterion selected. Two of the most useful criteria are the signal-to-noise ratio and the mean-square-error criteria. The latter criterion combined with the method of steepest descent leads to the Widrow-Hoff algorithm. This algorithm is often implemented in practical systems.

The adaptive noise canceller differs from classical adaptive systems primarily in that the estimate of the desired response is derived from a separate antenna, called the primary antenna, rather than from an internal generator. The sidelobe canceller is a special type of adaptive noise canceller. The adaptive notch filter, which removes periodic interference signals from wide-bandwidth desired signals, is another variation of the adaptive noise canceller.

Classical adaptive systems and the adaptive noise canceller sometimes exhibit unintentional cancellation of the desired signal. To preclude such a possibility, the constrained minimum power criterion can be used in the design of an adaptive system. The resulting system automatically limits the cancellation of the desired signal, while still adaptively filtering the interference.

2. SIDELOBE CANCELLER

An adaptive antenna system automatically monitors its output and adjusts its parameters accordingly. It does so in order to reduce the impact of interference that enters through the sidelobes, or possibly the mainlobe, of its antenna radiation pattern, while still allowing reception of an intended transmission. The design of an adaptive antenna system requires little *a priori* knowledge of the signal or interference characteristics.

The sidelobe canceller is a classic example of an adaptive antenna system. It not only is of practical importance, but also provides an introduction to the fundamental concepts of adaptive antenna systems.

Figure 1 shows a version of a sidelobe canceller. The primary and reference signals are outputs of two separate antennas or two different groups of elements in a single phased-array antenna. It is intended that the reference signal provide an estimate of the interference in the primary signal. After suitable processing, this estimate is subtracted from the primary signal. As a result, the interference, which may have entered through the sidelobes of the primary antenna, is reduced or eliminated by cancellation.

Ideally, the reference signal, $X_1(t)$, has a large interference component and a small desired signal

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PRIMARY ANTENNA

REFERENCE ANTENNA

BANDPASS FILTER

BANDPASS FILTER

BEAM STEERING NETWORK

BEAM STEERING NETWORK

MONITORING NETWORK

$X_0(t)$

$X_1(t)$

$X_2(t)$

$\lambda/4$

$W_1(t)$

$W_2(t)$

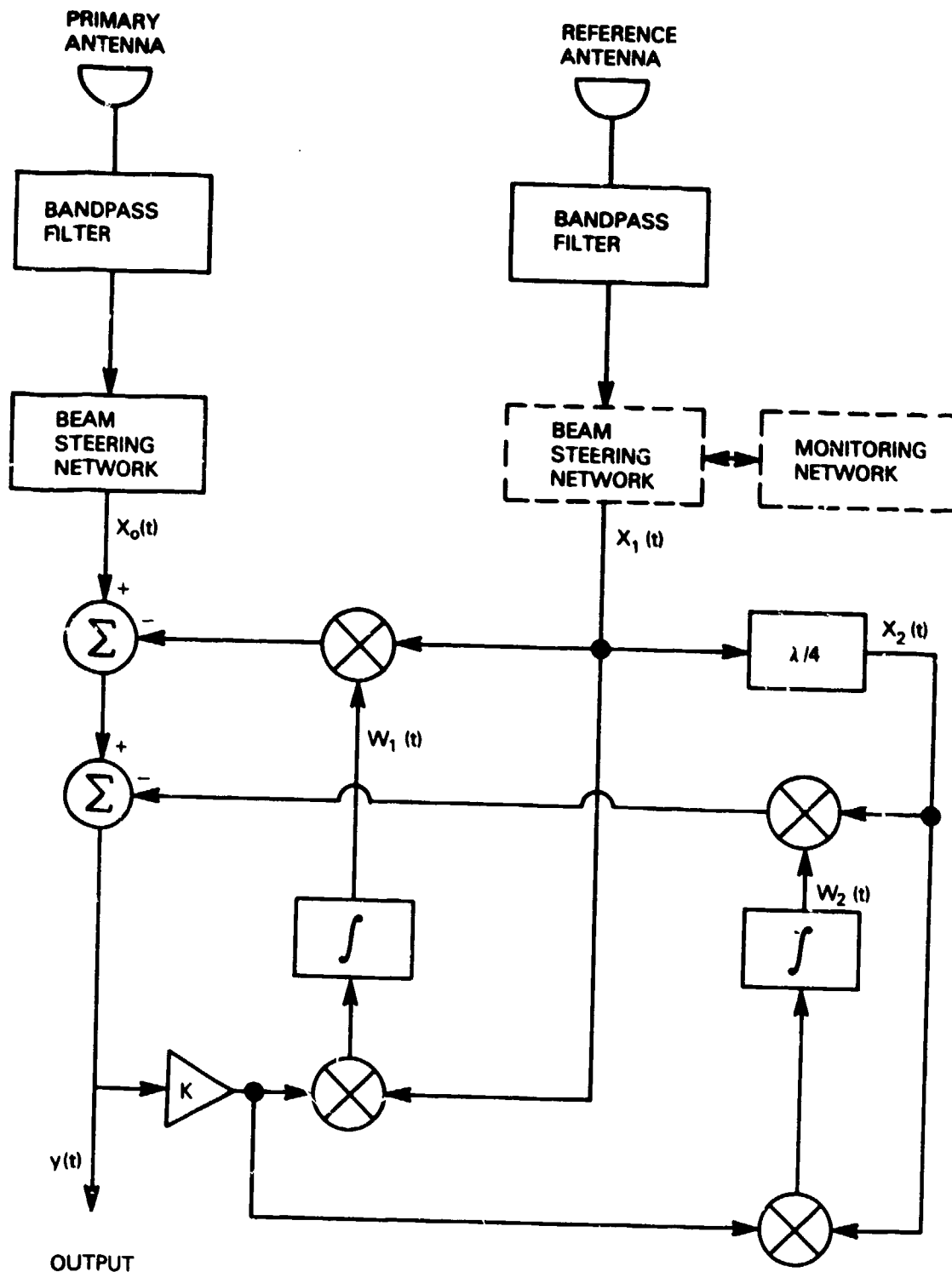$\int$

$\int$

K

$y(t)$

OUTPUT

Figure 1. Sidelobe canceller.

component, whereas $X_0(t)$ may have a much larger desired signal component. The quarter-wavelength delay shown in figure 1 produces a signal, $X_2(t)$, that is in phase quadrature with $X_1(t)$ (a quadrature hybrid could be used instead of the delay). The weight functions, $W_1(t)$ and $W_2(t)$, regulate the amounts of $X_1(t)$ and $X_2(t)$ that are subtracted from $X_0(t)$. The relative magnitudes of the weight functions determine the magnitude and phase of the total waveform that is subtracted from $X_0(t)$. If the magnitudes and phases of the interferer components of $X_0(t)$ and the total subtracted waveform are equal, the interference is cancelled and does not appear in the output, $y(t)$. If the cancellation is nearly complete, the weight functions are nearly constants; if it is not, the weight functions vary in such a way that the total subtracted waveform gradually becomes a facsimile of the interference component of $X_0(t)$.

In the following analysis, we make no assumptions about the directionality of the primary antenna beam or the reference antenna beam. However, it is highly desirable to have the primary beam point in the direction of the desired signal. The beam-steering network forms a beam in the appropriate direction by using various types of *a priori* information. If the antenna is part of a radar system, the information may be radar return characteristics. If the antenna is part of a communication system, the information may be the characteristics of a pilot signal transmitted along with the message signal. The antenna system then locks onto the pilot to form a beam in the direction of the transmitter.

2.1 Steady-State Operation

In general, the desired signal received by the primary antenna has the form

$$s(t) = A_s(t) \cos \left[\omega_0 t + \phi_s(t) \right] , \quad (1)$$

where ω_0 is the carrier frequency, and $A_s(t)$ and $\phi_s(t)$ are modulation functions. We assume that interference (which may be due to jamming), after passage through the bandpass filter of the primary branch, has the form

$$J(t) = A_j(t) \cos \left[\omega_1 t + \phi_j(t) \right] , \quad (2)$$

where ω_1 is the carrier frequency, and $A_j(t)$ and $\phi_j(t)$ are modulation functions. Neglecting the thermal noise, the output of the bandpass filter of the primary branch is

$$X_0(t) = A_s(t) \cos \left[\omega_0 t + \phi_s(t) \right] +$$

$$A_j(t) \cos \left[\omega_1 t + \phi_j(t) \right] . \quad (3)$$

Because of a possible difference in the radiation patterns between the two antennas, the signals received by the reference antenna may experience different amplifications than the same signals do when received by the primary antenna. Each signal has a different arrival time and, hence, a phase shift at the reference antenna relative to the same signal at the primary antenna. We assume that the antennas are close enough that the difference in arrival time of a signal at the two antennas is much less than the inverse signal bandwidth. Thus, the modulation functions are negligibly affected by this difference. We conclude that the output of the bandpass filter of the reference branch can be represented by

$$X_1(t) = C_1 A_s(t) \cos \left[\omega_0 t + \phi_s(t) + \theta_1(t) \right] +$$

$$C_2 A_j(t) \cos \left[\omega_1 t + \phi_j(t) + \theta_2(t) \right] . (4)$$

where θ_1 and θ_2 are phase angles, and C_1 and C_2 are real constants. If the sources of the desired signal and the interference are separated geometrically, then $\theta_1 \neq \theta_2$. If the sources are mobile, then θ_1 and θ_2 are functions of time. The magnitudes of the primary antenna gains in the directions of the desired signal and the interference are denoted by G_{ps} and G_{pj}, respectively. The magnitudes of the reference antenna gains in the directions of the desired signal and the interference are denoted by G_{rs} and G_{rj}, respectively. From these definitions, we have

$$C_1 = \left(\frac{G_{rs}}{G_{ps}} \right)^{1/2} , C_2 = \left(\frac{G_{rj}}{G_{pj}} \right)^{1/2} . (5)$$

The integrators are designed to integrate over the time interval $I = [t - T, t]$, where T is such that

$$\omega_o T \gg 1 \ . \tag{6a}$$

We assume that the bandpass filter is narrowband so that

$$\omega_o \gg 2\pi B \ . \tag{6b}$$

where B is the bandwidth of the bandpass filters in hertz. Thus, the bandwidth due to the message modulation is much less than the carrier frequency. For practical values of ω_o, the bandwidths associated with the time variations of θ_1 and θ_2 are also much less than ω_o.

The quarter-wavelength delay is designed to introduce a 90-degree phase shift in the intended transmission. Since $\omega_o - \omega_1 < 2\pi B$, equation (6b) indicates that this delay also introduces nearly a 90-degree phase shift in the interference, although it does not significantly affect the modulation waveforms. Thus,

$$X_2(t) = C_1 A_s(t) \sin\left[\omega_o t + \phi_s(t) + \theta_1(t)\right] +$$
$$C_2 A_j(t) \sin\left[\omega_1 t + \phi_j(t) + \theta_2(t)\right] \ . \tag{7}$$

The signal-to-interference ratio at the primary input is defined by

$$\rho_i = \frac{\frac{1}{T}\int_I s^2(u)du}{\frac{1}{T}\int_I J^2(u)du} =$$

$$\frac{\int_I A_s^2 \ du + \int_I A_s^2 \cos(2\omega_o u + 2\phi_s)du}{\int_I A_j^2 \ du + \int_I A_j^2 \cos(2\omega_1 u + 2\phi_j)du} \cdot \tag{8}$$

In general, ρ_i is a function of time. The steady state is defined to exist when this ratio is nearly constant over time intervals of duration T or longer.

When the steady state is reached, equations (6) imply that the second terms in the numerator and denominator are negligible compared to the first terms. Thus,

$$\rho_i \simeq \frac{\int_I A_s^2 \ (u)du}{\int_I A_j^2 \ (u)du} = \frac{G_{ps}}{G_{pj}}\rho_n \ . \tag{9}$$

where ρ_n is the ratio that would exist if the primary pattern were omnidirectional. Similarly, the signal-to-interference ratio at the reference input is found to be

$$\rho_r \simeq \left(\frac{C_1}{C_2}\right)^2 \rho_i = \frac{G_{rs}}{G_{rj}}\rho_n \ . \tag{10}$$

Although ρ_i, ρ_r, and ρ_n are functions of time in general, they are nearly constant during the steady state.

From figure 1, the output, $y(t)$, and the weighting functions, $W_1(t)$ and $W_2(t)$, are given by

$$y(t) = X_o(t) - W_1(t)X_1(t) - W_2(t)X_2(t) \ , \tag{11}$$

$$W_1(t) = K\int_I y(u)X_1(u)du \ , \tag{12}$$

$$W_2(t) = K\int_I y(u)X_2(u)du \ , \tag{13}$$

where the constant K is the gain of a linear amplifier. Substituting equation (11) into equation (12), we obtain

$$W_1(t) = K\int_I X_o(u)X_1(u)du -$$
$$K\int_I W_1(u)X_1^2(u)du -$$
$$K\int_I W_2(u)X_1(u)X_2(u)du \ . \tag{14}$$

We shall show that $W_1(t)$ and $W_2(t)$ are nearly constants over interval I if ρ_r, θ_1, and θ_2 are nearly constants over interval I. In this case, $W_1(t)$ and $W_2(t)$ can be removed outside the integrals with negligible error, so that

$$W_1(t) = K\int_I X_o(u)X_1(u)du -$$
$$KW_1(t)\int_I X_1^2(u)du -$$
$$KW_2(t)\int_I X_1(u)X_2(u)du \ . \tag{15}$$

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In a similar manner, we obtain

$$W_2(t) = K \int_I X_0(u)X_2(u)du - KW_2(t) \int_I X_2^2(u)du$$
$$- KW_1(t) \int_I X_1(u)X_2(u)du \ . \qquad (16)$$

Equations (6) and simple trigonometry show that

$$\int_I X_2^2(u)du \simeq \int_I X_1^2(u)du \qquad (17)$$

during the steady state. Using this approximation to solve equations (15) and (16) simultaneously yields

$$W_1(t) = \frac{V_1 V_3 - V_2 V_4}{V_1^2 - V_2^2} \qquad (18)$$

and

$$W_2(t) = \frac{V_1 V_4 - V_2 V_3}{V_1^2 - V_2^2} \ . \qquad (19)$$

In the above we define

$$V_1 = \frac{1}{K} + \int_I X_1^2(u)du \ , \qquad (20)$$

$$V_2 = \int_I X_1(u)X_2(u)du \ , \qquad (21)$$

$$V_3 = \int_I X_0(u)X_1(u)du \ , \qquad (22)$$

$$V_4 = \int_I X_0(u)X_2(u)du \ . \qquad (23)$$

Evaluating the integrals with equations (3), (4), and (7), and substituting equations (18) and (19) into equation (11) gives an expression for the output in terms of the desired signal and interference.

In most cases of interest, some simplification is possible. If T and K are sufficiently large, the first term on the right-hand side of equation (20) can be ignored. Substituting equations (4) and (7) into equation (21) shows that V_2 is very small during steady-state operation. Consequently, it is reasonable to neglect V_2 in equations (18) and (19). As a result of these approximations, we have

$$W_1(t) \simeq \frac{\int_I X_0(u)X_1(u)du}{\int_I X_1^2(u)du} \ , \qquad (24)$$

$$W_2(t) = \frac{\int_I X_0(u)X_2(u)du}{\int_I X_1^2(u)du} \ . \qquad (25)$$

Thus, the weighting functions are essentially normalized cross-correlation functions. From equations (3), (4), (6), and (7), we obtain

$$\int_I X_1^2(u)du = \int_I \left\{ \frac{C_1^2}{2} A_s(u) + \frac{C_2^2}{2} A_j^2(u) + \right.$$

$$C_1 C_2 A_s(u)A_j(u)$$

$$\left. \cos\left[(\omega_0 - \omega_1)u + \phi_s(u) - \phi_j(u) + \theta_1 - \theta_2\right] \right\} du \ , \qquad (26)$$

$$\int_I X_0(u)X_1(u)du = \int_I \left\{ \frac{C_1}{2} A_s^2(u) \cos \theta_1 + \right.$$

$$\frac{C_2}{2} A_j^2(u) \cos \theta_2 + \frac{C_1}{2} A_s(u)A_j(u)$$

$$\cos\left[(\omega_0 - \omega_1)u + \phi_s(u) - \phi_j(u) + \theta_1\right] +$$

$$\frac{C_2}{2} A_s(u)A_j(u)$$

$$\left. \cos\left[(\omega_0 - \omega_1)u + \phi_s(u) - \phi_j(u) - \theta_2\right] \right\} du \ . \qquad (27)$$

$$\int_I X_0(u)X_2(u)du = \int_I \left\{ \frac{C_1}{2} A_s^2(u) \sin \theta_1 + \right.$$

$$\frac{C_2}{2} A_j^2(u) \sin \theta_2 + \frac{C_1}{2} A_s(u)A_j(u)$$

$$\sin\left[(\omega_0 - \omega_1)u - \phi_s(u) - \phi_j(u) + \theta_1\right] -$$

$$\frac{C_2}{2} A_s(u) A_i(u)$$

$$\sin\left[(\omega_0 - \omega_1)u + \phi_s(u) - \phi_j(u) - \theta_2\right]\bigg\} \, du \quad . \tag{28}$$

In general, since the desired signal is unsynchronized with the interference, we expect that during steady-state operation,

$$\int_I s_1(u, \phi_1) \, J_1(u, \phi_2) \, du \simeq 0 \quad , \tag{29}$$

where $s_1(u, \phi_1)$ is the desired signal with an arbitrary phase shift of ϕ_1, and $J_1(u, \phi_2)$ is the interference signal with an arbitrary phase shift of ϕ_2. If equation (29) is valid, the desired signal and interference are said to be uncorrelated in the time-average sense.

Equation (29) implies that only the first two terms of equations (26) through (28) need be retained. Using equations (9) and (10), equations (24) through (28) yield

$$W_1 = \frac{C_1 \cos \theta_2 + C_2 \rho_r \cos \theta_1}{C_1 C_2 (1 + \rho_r)} \quad , \tag{30}$$

$$W_2 = \frac{C_1 \sin \theta_2 + C_2 \rho_r \sin \theta_1}{C_1 C_2 (1 + \rho_r)} \quad . \tag{31}$$

These equations indicate that W_1 and W_2 are approximately constant over interval I if ρ_r, θ_1, and θ_2 are approximately constant over interval I. Thus, the initial assumption that the weighting functions are constant has been verified as consistent with the other approximations made in deriving these equations.

We now substitute into equation (11) and employ trigonometric identities. Using equations (9) and (10), the final result is

$$(1 + \rho_r) \, y(t) = A_s(t) \cos\left[\omega_0 t + \phi_s(t)\right] -$$

$$\frac{1}{\alpha} A_s(t) \cos\left[\omega_0 t + \phi_s(t) + \theta_1 - \theta_2\right] +$$

$$\rho_r A_j(t) \cos\left[\omega_1 t + \phi_j(t)\right] -$$

$$\rho_r \, \alpha \, A_j(t) \cos\left[\omega_1 t + \phi_j(t) - \theta_1 + \theta_2\right] \tag{32}$$

where

$$\alpha = \left(\frac{G_{ps} G_{rj}}{G_{pj} G_{rs}}\right)^{1/2} \quad . \tag{33}$$

The signal-to-interference ratio at the output of the sidelobe canceller, ρ_o, is calculated as the ratio of the average power over interval I in the first two terms to the average power over interval I in the last two terms of the right side of equation (32). If $y(t) \neq 0$, the result is the remarkably simple formula,

$$\rho_o = \frac{1}{\rho_r} \quad . \tag{34}$$

Thus, the interference component of the primary signal has been nearly cancelled if $\rho_r \ll 1$. We conclude that the output signal distortion is small when the signal power at the reference antenna is relatively low. An important approximation made in deriving equations (32) and (34) is that the thermal noise is negligible.

If the interference source is almost directly behind the source of the desired signal, then $\alpha \simeq 1$ and $\theta_1 \simeq \theta_2$. Since equation (32) indicates that $y(t) \simeq 0$, the output signal is buried in the thermal noise. We conclude that the sidelobe canceller is ineffective in this case.

A strong desired signal in the reference channel can induce cancellation of the desired signal at the output. Specifically, if $\rho_i \geq \rho_r \geq 1$, equation (34) gives $\rho_o \leq \rho_i$. Consequently, the adaptive system causes a performance degradation relative to the performance of the primary antenna operating alone. Equation (10) indicates that designing the system so that $G_{rs} \ll G_{rj}$ is helpful in ensuring small values of ρ_r. Thus, a reference antenna with a beam that points in the direction of the interference is highly desirable. One possible implementation is to use a beam-steering network that enables a reference beam to search for interfering signals of high power or special characteristics.

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Since the direction of the interference is usually unknown *a priori*, and since there may be two or more spatially separated interference sources present, $G_{r_3} << G_{r_j}$ might be achieved if we deploy several reference antennas, each with a directional beam, as shown schematically in figure 2. The reference beams point in various directions rela-

small that $\rho_r >> 1$. To prevent this disaster, the reference input can be monitored and its propagation blocked unless its amplitude or power exceeds a fixed threshold. Thus, the adaptive mechanism is disabled unless the interference is strong enough to warrant its use.
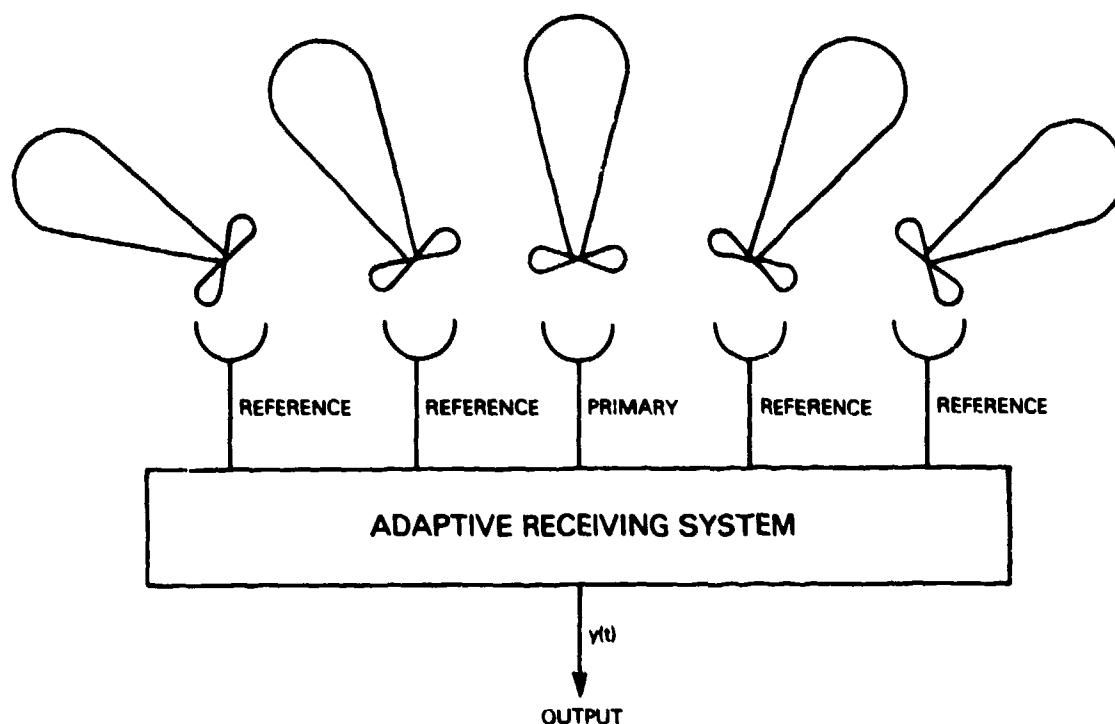

Figure 2. Multiple sidelobe canceller.

tive to the primary beam. The output of each reference antenna feeds a cancellation network similar to that of figure 1. A configuration of this type is known as a multiple sidelobe canceller since it provides the possibility of cancelling multiple interfering signals entering at two or more different points of the sidelobes of the primary pattern. A less expensive multiple sidelobe canceller uses omnidirectional reference antennas.[1]

Even if $G_{r_3} << G_{r_j}$, serious cancellation of the desired signal can occur if the interference is so

If the reference antenna is disconnected and an unmodulated carrier is injected as $X_1(t)$, the adaptive antenna system behaves as a notch filter at the carrier frequency. To demonstrate this effect, we set $C_1 = 0$ and assume that $A_j(t)$ and $\phi_j(t)$ are constants in equations (2) and (4). If equation (29) is valid, we obtain $y(t) = s(t)$, indicating complete cancellation of the interfering carrier. However, equation (29) is not valid if $A_s(t)$ and $\phi_s(t)$ are constants and $\omega_0 = \omega_1$. In this case, a straightforward but tedious calculation using equations (2), (11), and (24) through (28) gives $y(t) \simeq 0$, indicating complete cancellation of the primary input.

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[1] B. D. Steinberg, *Principles of Aperture and Array System Design*, Wiley (1976).

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2.2 Adaptive Null Steering

We can interpret the operation of the adaptive antenna system in terms of adaptive null steering. Since we are not seeking to obtain new results, but only to interpret old ones, we simplify the mathematics by assuming that $\alpha \gg 1$. Thus, the second term of equation (32) is negligible compared to the first, and the third term is negligible compared to the fourth. Decomposing the first and fourth terms yields

$$y(t) \simeq \left[s(t) + J_1(t) \right] -$$

$$\left[\frac{\rho_r}{1 + \rho_r} \, s(t) + \left(1 + \frac{\rho_r \alpha}{1 + \rho_r} \right) J_1(t) \right] \, ,$$

$$\alpha \gg 1 \, , \qquad (35)$$

where

$$J_1(t) = A_j(t) \cos \left[\omega_1 t + \phi_j(t) - \theta_1 + \theta_2 \right] \qquad (36)$$

is a phase-shifted version of $J(t)$. The first bracketed term on the right side of equation (35) can be interpreted as the response due to the primary antenna with gains G_{ps} and G_{pj} in the directions of the desired signal and interference, respectively. The second bracketed term can be interpreted as the response due to an equivalent pattern, the output of which is subtracted from the primary output to give $y(t)$.

With this interpretation, the equivalent cancellation pattern or adaptive beam has gains

$$G_s' = \left(\frac{\rho_r}{1 + \rho_r} \right)^2 G_{ps} \, , \quad \alpha \gg 1 \quad . \quad (37)$$

$$G_j' = \left(1 + \frac{\rho_r \alpha}{1 + \rho_r} \right)^2 G_{pj} \, , \quad \alpha \gg 1 \, , (38)$$

in the directions of the desired signal and interference, respectively. The equivalent overall pattern of the adaptive antenna system provides the gains

$$G_s'' = \left(\sqrt{G_{ps}} - \sqrt{G_s'} \right)^2 = \left(\frac{1}{1 + \rho_r} \right)^2 G_{ps} \, ,$$

$$\alpha \gg 1 \, , \qquad (39)$$

$$G_j'' = \left(\sqrt{G_{pj}} - \sqrt{G_j'} \right)^2 = \left(\frac{\rho_r \alpha}{1 + \rho_r} \right)^2 G_{pj} \, ,$$

$$\alpha \gg 1 \, , \qquad (40)$$

in the directions of the desired signal and interference, respectively. If ρ_r is small, equation (39) gives $G_s'' \simeq G_{ps}$. If ρ_r is so small that $\rho_r \alpha \ll 1$, equation (40) indicates that G_j'' is much smaller than G_{pj}. Thus, an approximate null can be created in the direction of the interference. For this reason, the action of an adaptive antenna system is sometimes called "null steering." A pictorial representation is given in figure 3.
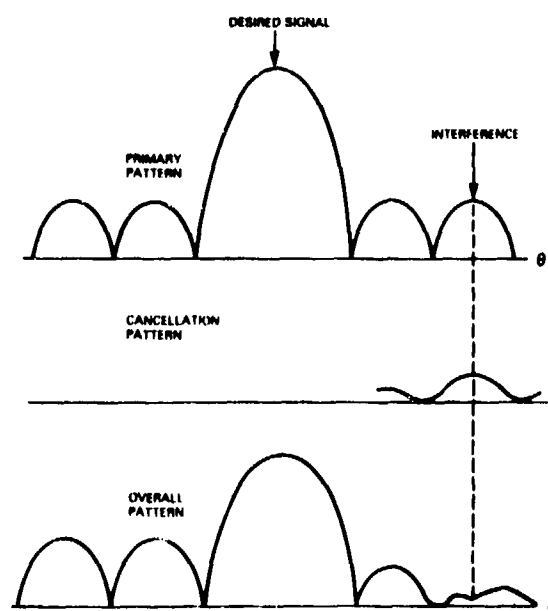

Figure 3. Adaptive beam forming and null steering.

An analysis based upon the Wiener-Hopf equation, which is derived in section 3.3, shows that if the narrowband condition given by equation (6b) is not satisfied, then the performance of the sidelobe canceller of figure 1 deteriorates.[2] Consequently, adaptive systems intended to handle wide-bandwidth signals must use more elaborate processing, such as that provided by delay lines with multiple taps, which are discussed in section 3.

Although the major features of the sidelobe canceller have been determined, the theoretical motivation for the configuration of figure 1 has not been discussed. For this purpose, and to explore alternative configurations, we consider the classical theory of adaptive antenna systems.

3. CLASSICAL THEORY

An adaptive antenna system consists of an antenna array, fixed processing elements, and adaptive elements. Its purpose is to remove externally generated interference from a receiver signal. The adaptive elements have one of the forms shown in figure 4. In figure 4(a), the weight adjustment mechanism responds to the output; in figure 4(b), it responds to the error, which is the difference between the desired response and the output. Ideally, the desired response, d, is the received signal minus the interference and noise. In practice, d is a signal with the general characteristics, but not the detailed structure, of the signals the antenna system is attempting to receive. The forms in figure 4 are not necessarily optimal, but are used because of their simplicity and compatibility with computer-controlled systems. The inputs, $X_1, X_2, \ldots, X_N$, are derived from fixed processing elements and may be either continuous or discrete-time signals. The inputs are applied to weights $W_1, W_2, \ldots, W_N$, which are continually adjusted as a function of the output or the error. The basic adaptive elements can be used in a variety of different systems.

[2] W. E. Rodgers and R. J. Compton, Adaptive Array Bandwidth with Tapped Delay-Line Processing, IEEE Trans. Aerospace Electron. Syst., AES-15 (January 1979), 21.

The inputs may be derived from a tapped delay line, as shown in figure 5. Each of the K antenna outputs is filtered, delayed, and then applied to a line consisting of L tap points and $L - 1$ ideal time delays of δ seconds each. With $N = KL$, the adaptive part of figure 5 is a special case of figure 4.

Each input of figure 4 can be the discrete Fourier transform at a specific frequency, say f_i, of an antenna output. With this interpretation for figure 4(b), the desired response is the discrete Fourier transform at f_i of the desired waveform. A complete frequency-domain array-processing filter feeds K of the adaptive elements — one for each of K discrete frequencies, as illustrated in figure 6. This filter might be attractive if the desired waveform is a complicated function of time but has a simple Fourier transform.

To develop adaptive algorithms, we first determine the optimal fixed weights. Then, adaptive algorithms can be determined that yield weights converging to the optimal fixed weights.

The derivation of the optimal fixed weights depends upon the specification of a performance criterion or estimation procedure and the modeling of the signals received by the antennas. For fixed weights, the optimization problem is illustrated in figure 7. The input and weight vectors are

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ . \\ X_N \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} W_1 \\ W_2 \\ . \\ . \\ . \\ W_N \end{bmatrix} . \quad (41)$$

The input is a function of the desired signal, interference, and thermal noise. The scalar output is

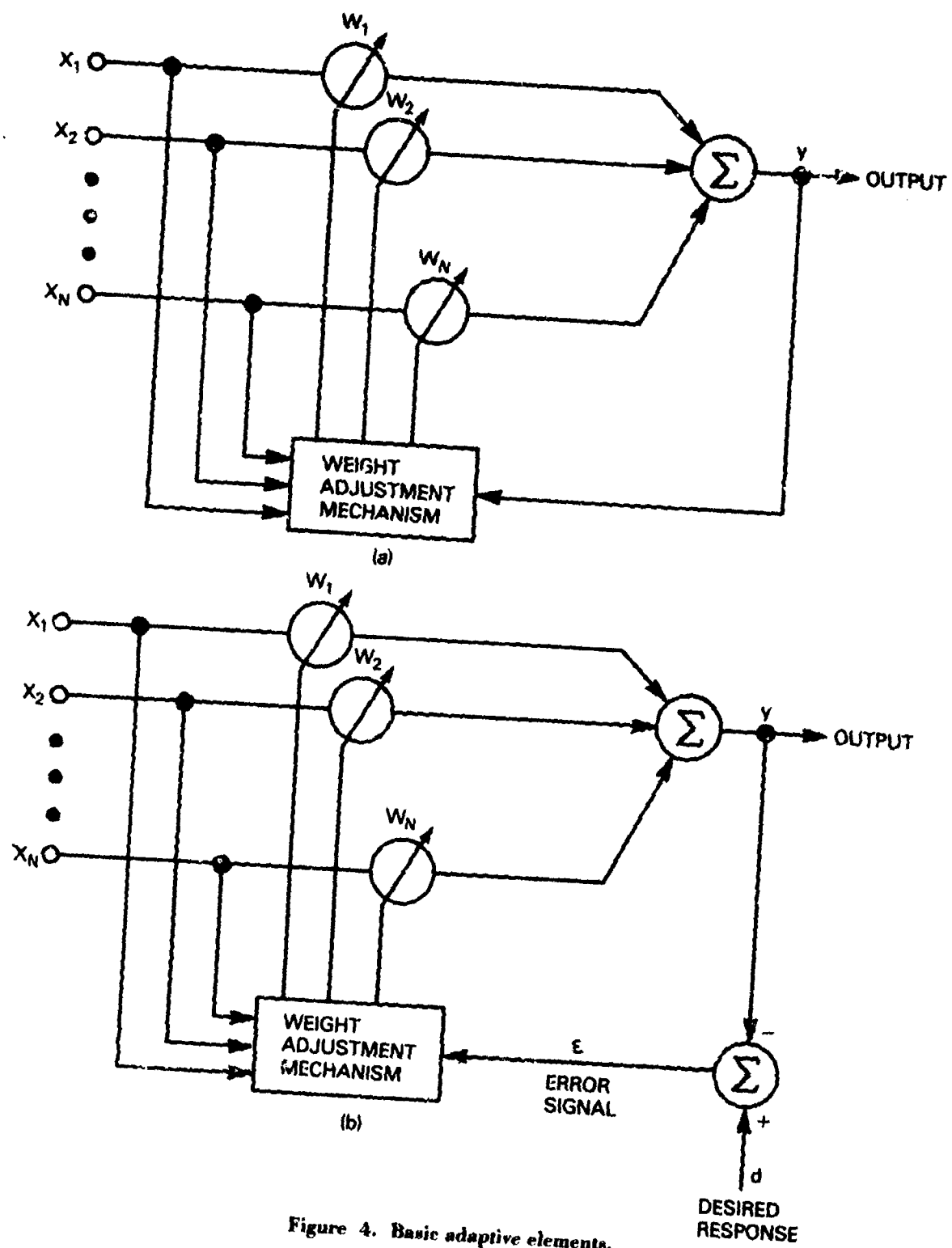$$y = \mathbf{W}^T \mathbf{X} . \quad (42)$$

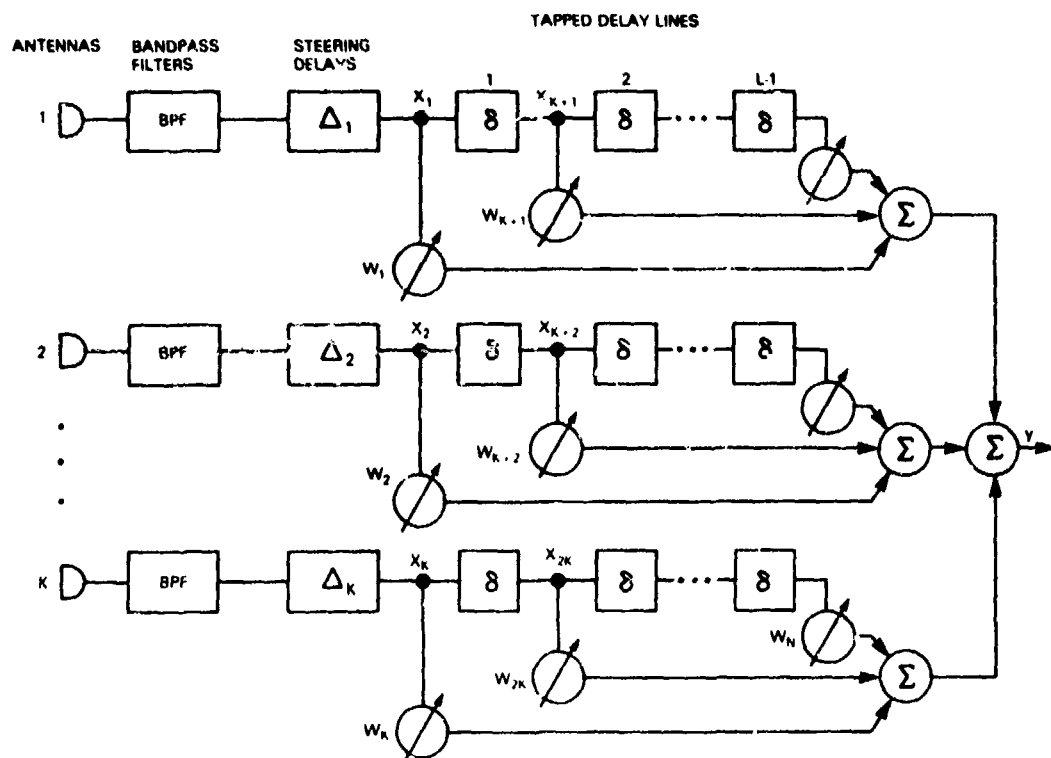13

Figure 4. Basic adaptive elements.

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TAPPED DELAY LINES

Figure 5. General form of tapped delay-line array-processing filter.
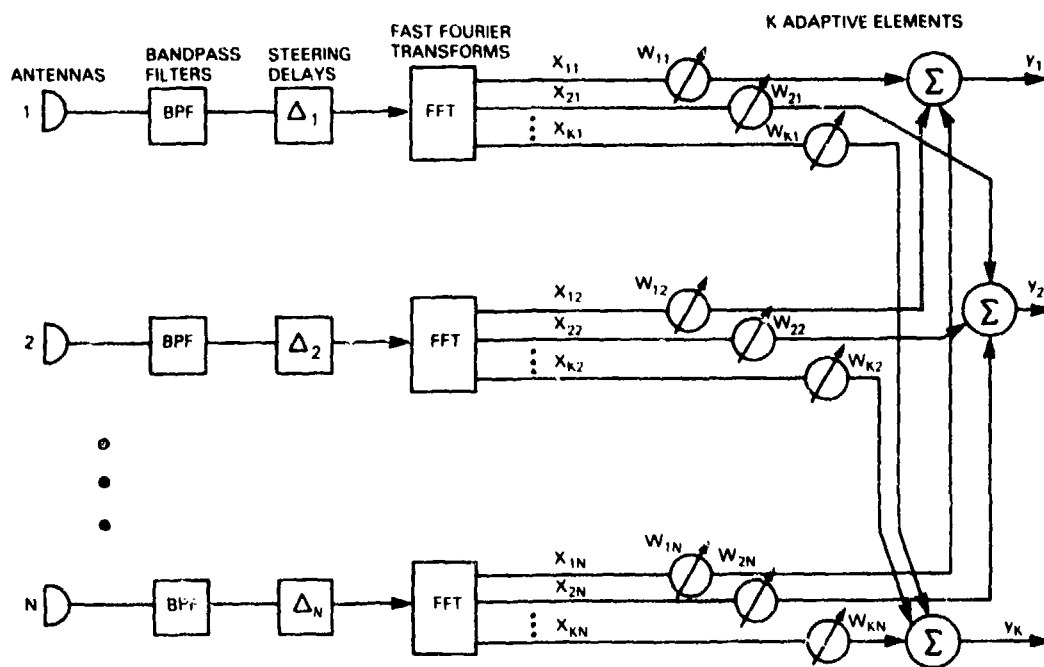
Figure 6. General form of frequency-domain array-processing filter.

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$$Y = W^T X$$

ESTIMATE OF
DESIRED RESPONSE

Figure 7. Optimization problem.

where the superscript T denotes the transpose operation. The output is an estimate of the desired response, d. The filter or combiner of figure 7 is linear because the weights are fixed. Since their characteristics change in response to changes in the characteristics of their inputs, adaptive systems are intrinsically nonlinear. However, adaptive systems can be designed that approximate the optimal linear filter after a sufficient amount of adaption.

Several different estimators of d can be implemented by the linear filter of figure 7. If the received vector, X, is assumed to have a Gaussian distribution, maximum likelihood estimation of the desired signal components of the received waveform leads to an estimator that is a linear function of X. However, the Gaussian assumption is unwarranted when interference is present. Estimators that depend only upon the second-order moments of X can be derived using performance criteria based upon the signal-to-noise ratio or the mean-square error of the filter output. Consequently, maximum likelihood estimation is not pursued further in this paper. Griffiths [3] compares the maximum likelihood filter and the Wiener filter, which is derived below.

3.1 Signal-to-Noise Ratio Criterion

A reasonable design criterion for adaptive radar systems is the maximization of the probability of detection for fixed probability of false alarm.[4] This maximization has been shown to be equivalent to a maximization of a generalized signal-to-noise ratio,

which is itself an intuitively appealing design criterion.

Consider the maximization of the signal-to-noise ratio at the output of the adaptive element of figure 4a.[5] By noise, we mean the combined thermal noise and externally generated interference. We assume that the input vector, X, is the sum of two components,

$$X = H d + n , \qquad (43)$$

where H is a vector of constants,

$$n = \begin{bmatrix} n_1 \\ n_2 \\ . \\ . \\ . \\ n_N \end{bmatrix} \qquad (44)$$

is a vector of noise components that are derived from stationary processes, and d is the deterministic desired signal. We treat all vector components and scalars as complex variables. Consequently, our results apply to adaptive elements fed by the system of figure 6 and to narrowband systems for which d is the complex envelope representation of the desired signal.

The component of the output, y, that is due to the desired signal is

$$y_s = W^T H d , \qquad (45)$$

[3] L. J. Griffiths, A Comparison of Multidimensional Wiener and Maximum-likelihood Filters for Antenna Arrays, Proc. IEEE (letters), 55, (November 1967), 2045.

[4] L. E. Brennan and I. S. Reed, Theory of Adaptive Radar, IEEE Trans. Aerospace Electron. Syst. AES-9 (March 1973), 237.

[5] S. P. Applebaum, Adaptive Arrays, IEEE Trans. Antennas Prop., AP-26 (September 1976), 585.

and the component of y that is due to the noise is

$$y_n = \mathbf{W}^T \mathbf{n} = \mathbf{n}^T \mathbf{W} . \qquad (46)$$

At a single instant of time, the signal power at the output is

$$R_s = |y_s|^2 = |\mathbf{W}^T \mathbf{H} d|^2 . \qquad (47)$$

Let a prime denote the conjugate transpose operation and an asterisk denote the conjugate operation. The mean noise power in the output is

$$R_n = E[(\mathbf{n}^T \mathbf{W})'(\mathbf{n}^T \mathbf{W})] = \mathbf{W}' \mathbf{R}_{NN} \mathbf{W} , \qquad (48)$$

where $E[x]$ is the expected value of x and $\mathbf{R}_{NN} = E[\mathbf{n}^* \mathbf{n}^T]$ is the correlation matrix of the noise. The signal-to-noise ratio is

$$P = \frac{R_s}{R_n} = \frac{(\mathbf{W}^T \mathbf{H})'(\mathbf{W}^T \mathbf{H})|d|^2}{\mathbf{W}' \mathbf{R}_{NN} \mathbf{W}} \qquad (49)$$

Since $R_n > 0$ in practical cases, $\mathbf{R}_{NN}$ is positive definite. From its definition, $\mathbf{R}_{NN}$ is Hermitian.

A positive definite Hermitian matrix has positive eigenvalues. Thus, it can be diagonalized to the identity matrix. We define the nonsingular $N \times N$ matrix,

$$\mathbf{D} = \begin{bmatrix} \dfrac{\mathbf{e}_1}{\sqrt{\lambda_1}} & \dfrac{\mathbf{e}_2}{\sqrt{\lambda_2}} & \cdots & \dfrac{\mathbf{e}_N}{\sqrt{\lambda_N}} \end{bmatrix} , \qquad (50)$$

where the λ_i are the eigenvalues and the $\mathbf{e}_i$ are the corresponding orthonormal eigenvectors. Straightforward matrix algebra yields

$$\mathbf{I} = \mathbf{D}' \mathbf{R}_{NN} \mathbf{D} , \qquad (51)$$

where $\mathbf{I}$ is the identity matrix.

Combining this result with equation (48) gives

$$R_n = \mathbf{W}' (\mathbf{D}')^{-1} \mathbf{D}' \mathbf{R}_{NN} \mathbf{D} \mathbf{D}^{-1} \mathbf{W} =$$

$$(\mathbf{D}^{-1} \mathbf{W})'(\mathbf{D}^{-1} \mathbf{W}) = \|\mathbf{D}^{-1} \mathbf{W}\|^2 , \qquad (52)$$

where $\|x\|^2 = x' x$ is the Euclidean norm of the vector x. Using the Cauchy-Schwarz inequality, equation (47) yields

$$R_s = |\mathbf{W}^T (\mathbf{D}^T)^{-1} \mathbf{D}^T \mathbf{H} d|^2 =$$

$$|(\mathbf{D}^{-1} \mathbf{W})^T \mathbf{D}^T \mathbf{H} d|^2 =$$

$$|(\mathbf{D}^{-1} \mathbf{W})' \mathbf{D}' \mathbf{H}^*|^2 |d|^2 \le$$

$$\|\mathbf{D}^{-1} \mathbf{W}\|^2 \|\mathbf{D}' \mathbf{H}^*\|^2 |d|^2 . \qquad (53)$$

Equality in this equation is attained if

$$\mathbf{D}^{-1} \mathbf{W} = c \mathbf{D}' \mathbf{H}^* , \qquad (54)$$

where c is an arbitrary constant. Substituting equations (52) and (53) into equation (49), it follows that

$$P \le \|\mathbf{D}' \mathbf{H}^*\|^2 |d|^2 . \qquad (55)$$

The maximum value of P is attained if equation (54) is satisfied. Thus, the optimal choice of $\mathbf{W}$ is

$$\mathbf{W}_0 = c \mathbf{D} \mathbf{D}' \mathbf{H}^* . \qquad (56)$$

Using equation (51), this relation becomes

$$\mathbf{W}_0 = c \mathbf{R}_{NN}^{-1} \mathbf{H}^* . \qquad (57)$$

The value of P corresponding to optimal weights is

$$P_0 = \|\mathbf{D}' \mathbf{H}^*\|^2 |d|^2 =$$

$$\mathbf{H}^T \mathbf{D} \mathbf{D}' \mathbf{H}^* |d|^2 = \mathbf{H}^T \mathbf{R}_{NN}^{-1} \mathbf{H}^* |d|^2 . \qquad (58)$$

In order to implement equation (57), we must know the values of the components of $\mathbf{H}$. Since

adaptive systems have more than one antenna (or array element), this knowledge implies that the direction of arrival of the desired signal is known.

3.2 Mean-Square Error Criterion

The most widely used method of estimation is based on the minimization of the mean-square error.[6] Instead of assuming a known direction of arrival, we assume that a facsimile of the desired response is supplied. We assume that X and d are derived from stationary stochastic processes. Similar results can be obtained by modeling X and d as continuous-time or discrete-time deterministic variables and minimizing the time- or sample-averaged square error.

Because it is convenient and because the results are usually applied to systems of the form of figure 5, we assume that all scalars and matrix components are real. Referring to figure 4(b), the difference between the desired response and the output is the error signal,

$$\varepsilon = d - \mathbf{W}^T \mathbf{X} . \tag{59}$$

We square both sides of this equation and take the expected value. Regarding $\mathbf{W}$ as fixed, and noting that $\mathbf{W}^T \mathbf{X} = \mathbf{X}^T \mathbf{W}$, the mean-square error is

$$E\left[\varepsilon^2\right] = E\left[(d - \mathbf{W}^T\mathbf{X})^2\right] =$$

$$E\left[d^2\right] + \mathbf{W}^T\mathbf{R}_{xx}\mathbf{W} - 2\mathbf{W}^T\mathbf{R}_{xd} , \tag{60}$$

where

$$\mathbf{R}_{xx} = E\left[\mathbf{X}\mathbf{X}^T\right] \tag{61}$$

is the symmetric correlation matrix of $\mathbf{X}$, and

$$\mathbf{R}_{xd} = E\left[\mathbf{X}d\right] \tag{62}$$

is a vector of cross correlations between the input signals and the desired response. Taking the

[6] R. Widrow et al, Adaptive Antenna Systems, Proc. IEEE, 55 (December 1967), 2143.

gradient of equation (60) with respect to $\mathbf{W}$ yields

$$\nabla E\left[\varepsilon^2\right] = 2\mathbf{R}_{xx}\mathbf{W} - 2\mathbf{R}_{xd} . \tag{63}$$

Setting the gradient equal to zero, we obtain a necessary condition for the optimal weight vector $\mathbf{W}_0$. In practice, $\mathbf{R}_{xx}$ is positive definite. Thus, it is nonsingular so that we obtain

$$\mathbf{W}_0 = \mathbf{R}_{xx}^{-1}\mathbf{R}_{xd} . \tag{64}$$

This equation is the Wiener-Hopf equation for the optimal weight vector. The associated linear filter is called the Wiener filter. To show that $\mathbf{W}_0$ produces the minimum mean-square error, we set $\mathbf{W} = \mathbf{W}_0$ in equation (60) to obtain the mean-square error corresponding to $\mathbf{W}_0$,

$$E_R = E\left[d^2\right] - \mathbf{W}_0^T\mathbf{R}_{xx}\mathbf{W}_0 =$$

$$E\left[d^2\right] - \mathbf{R}_{xd}^T\mathbf{R}_{xx}^{-1}\mathbf{R}_{xd} . \tag{65}$$

Substituting equation (65) into equation (60) and using equation (64) to eliminate $\mathbf{R}_{xd}$, we may express the mean-square error in the form.

$$E\left[\varepsilon^2\right] = E_R + \left(\mathbf{W} - \mathbf{W}_0\right)^T \mathbf{R}_{xx}\left(\mathbf{W} - \mathbf{W}_0\right) . \tag{66}$$

Since $\mathbf{R}_{xx}$ is positive definite, this expression shows that $\mathbf{W}_0$ is a unique optimal weight vector and E_R is the minimum mean-square error. Equation (66) describes a multi-dimensional quadratic function of the weights that can be visualized in two dimensions as a bowl-shaped surface. The purpose of adaptation is to continually seek the bottom of the bowl.

3.3 Steepest Descent

In the implementation of either equation (57) or equation (64), the presence of interference means that the correlation matrices, $\mathbf{R}_{NN}$ or $\mathbf{R}_{xx}$, are unknown. For this reason, approximations of the equations must be determined. Since the computational difficulties of the matrix inversions are considerable when the number of weights is large or time-varying signal statistics require frequent

computations, most of the approximations developed involve recursion relations and gradient or random search techniques.[7] However, for sampled data systems, direct implementation of the equations after suitable approximations of the matrix elements has been shown to exhibit rapid convergence in certain circumstances.[8]

Because the theory has been extensively investigated and it has often been implemented, we consider the approximation of equation (64) by the method of steepest descent. The approximation of equation (57) implies a similar adaptive weight adjustment mechanism.[8] We shall treat $\mathbf{X}$, $\mathbf{W}$, and d as discrete-time, sampled-value variables and use the index k to denote a particular sampling instant or adaptation cycle. The results for continuous-time systems are analogous.

In the method of steepest descent, the weight vector is changed along the direction of the negative gradient of the mean-square error. Using equation (63), we obtain

$$\mathbf{W}(k + 1) = \mathbf{W}(k) - \mu \nabla \mathrm{E}\left[\varepsilon^2\right] =$$

$$\mathbf{W}(k) - 2\mu \left[\mathbf{R}_{xx}\mathbf{W}(k) - \mathbf{R}_{xd}\right] , \quad (67)$$

where the scalar constant μ controls the rate of convergence and stability. The adaptation cycle begins with an arbitrary initial weight. As it stands, this method eliminates the need to compute the inverse of $\mathbf{R}_{xx}$, but still requires approximation of $\mathbf{R}_{xx}$ and $\mathbf{R}_{xd}$.

The Griffiths algorithm[9] results if $\mathbf{R}_{xx}$ is replaced by $\mathbf{X}(k)\mathbf{X}^T(k)$ and $\mathbf{R}_{xd}$ is directly estimated, that is, we use

$$\mathbf{W}(k + 1) = \mathbf{W}(k) - 2\mu \left[\mathbf{X}(k)\mathbf{X}^T(k)\mathbf{W}(k) - \hat{\mathbf{R}}_{xd}\right]$$

$$= \mathbf{W}(k) - 2\mu\left[\mathbf{X}(k)y(k) - \hat{\mathbf{R}}_{xd}\right] , \quad (68)$$

where $\hat{\mathbf{R}}_{xd}$ is an estimate of $\mathbf{R}_{xd}$. The problem with this algorithm is that an accurate estimate of $\mathbf{R}_{xd}$ is difficult if the amplitude of the desired signal is unknown.

The Widrow-Hoff algorithm,[6] also known as the least-mean-square (LMS) algorithm, approximates $\mathbf{R}_{xd}$ by $\mathbf{X}(k)d(k)$ so that

$$\mathbf{W}(k + 1) = \mathbf{W}(k) - 2\mu\left[\mathbf{X}(k)y(k) - \mathbf{X}(k)d(k)\right]$$

$$= \mathbf{W}(k) - 2\mu\varepsilon(k)\mathbf{X}(k) . \quad (69)$$

This iteration rule states that the next weight vector is obtained by adding to the present weight vector the input vector scaled by the value of the error. It can be shown that for a sufficiently small value of μ, the mean of the weight vector converges to the optimal value given by the Wiener-Hopf equation. Figure 8 shows a block-diagram representation of equation (69) for one component of the weight vector.

In analog implementations of continuous systems, equation (69) becomes the differential equation,

$$\frac{d}{dt}\mathbf{W}(t) = -2\mu\varepsilon(t)\mathbf{X}(t) . \quad (70)$$

Equivalently, if $\mathbf{W}(0) = 0$,

$$\mathbf{W}(t) = -2\mu \int_0^t \varepsilon(\tau)\mathbf{X}(\tau)d\tau . \quad (71)$$

Figure 9 shows a realization of one component of this equation.

One potential problem with the Widrow-Hoff algorithm can be explained in terms of the overall equivalent radiation pattern of the antenna array. Adaptation causes the array to form a beam in the direction of the desired signal, while reducing or nulling the array response in the direction of the interference. If the desired signal is not transmitted, the adaptation may proceed to create a reduced response in the direction of this signal so that

[4] L. E. Brennan and I. S. Reed, Theory of Adaptive Radar, IEEE Trans. Aerospace Electron. Syst., AES-9 (March 1973), 237.

[7] B. Widrow and J. M. McCool, A Comparison of Adaptive Algorithms Based on the Methods of Steepest Descent and Random Search, IEEE Trans. Antennas Prop., AP-24 (September 1976), 615.

[8] I. S. Reed et al, Rapid Convergence Rates in Adaptive Arrays, IEEE Trans. Aerospace Electron. Syst. IES-10 (November 1974), 853.

[9] L. J. Griffiths, A Simple Adaptive Algorithm for Real-Time Processing in Antenna Arrays, Proc. IEEE, 57 (October 1969), 1696.

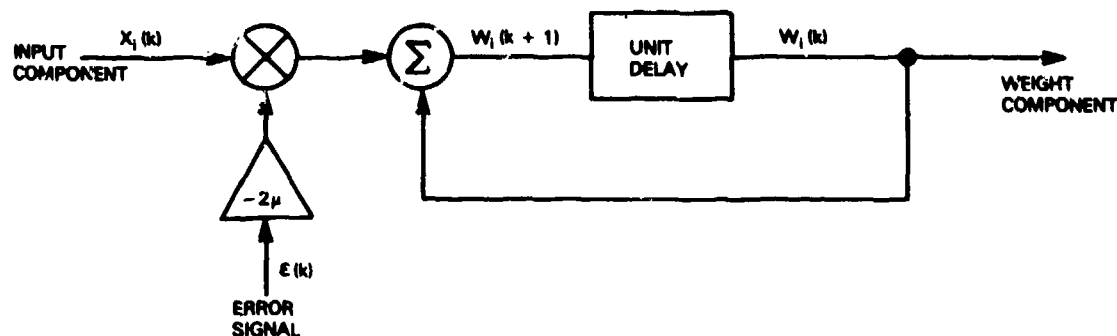[6] B. Widrow et al, Adaptive Antenna Systems, Proc. IEEE, 55 (December 1967), 2143.

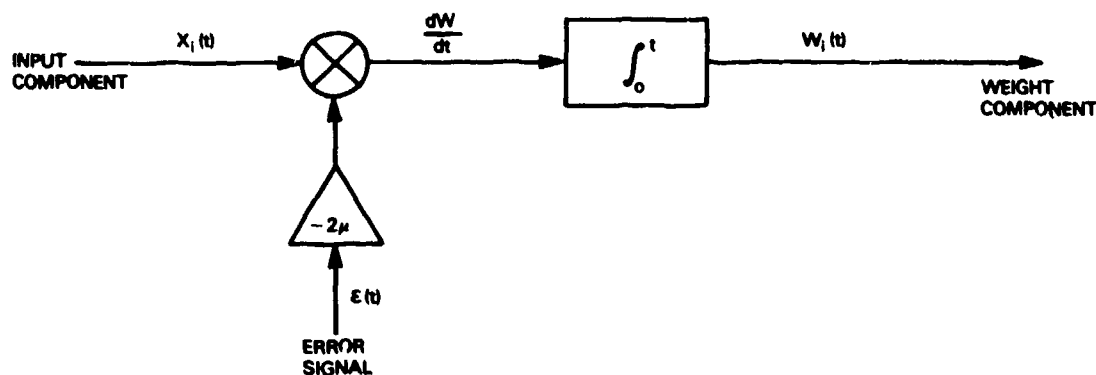**Figure 8. Digital realization of Widrow-Hoff algorithm.**

Figure 9. Analog realization of Widrow-Hoff algorithm.

communication is hindered or disabled when transmission is resumed. To alleviate this problem, a pilot signal may be used.

A pilot signal is a vector, each component of which is an estimate of the desired signal that would appear at an antenna output if no interference or noise were present. In other words, the pilot signal is constructed to have spectral and directional characteristics similar to those of the incoming signal of interest. Two different algorithms that use the pilot signal have been proposed. In the two-mode algorithm, adaptation alternates between adapting only on the pilot signal and adapting only on the signals actually received by the array. The one-mode algorithm adapts on the sum of the pilot signal and the received signal. Details of the performance of the pilot signal method can be found in the literature.[6, 9]

[6] B. Widrow et al, Adaptive Antenna Systems, Proc. IEE, 55 (December 1967), 2143.

[9] L. J. Griffiths, A Simple Adaptive Algorithm for Real-Time Processing in Antenna Arrays, Proc. IEEE, 57 (October 1969), 1696.

The desired response, d, is required at each iteration of equation (69). An estimate of the desired response, denoted by $\hat{d}$, can be generated for some types of communications by feedback systems of the forms shown in figure 10. For effective operation, $\hat{d}$ does not have to be a perfect replica of the desired response; if it were, we would not need the adaptive antenna system. However, the Wiener-Hopf equation, which we would like to approximately satisfy, indicates that $\hat{d}$ should be such that

$$\mathbf{R}_{x\hat{d}} \simeq \mathbf{R}_{xd} . \qquad (72)$$

Stable operation is achieved only if $\hat{d}$ is independent of the amplitude of the component proportional to the desired signal in the array output. If it is not, the array weights may increase indefinitely or decrease to zero. The presence of the limiter in figure 10(b) controls the amplitude of $\hat{d}$. The delay
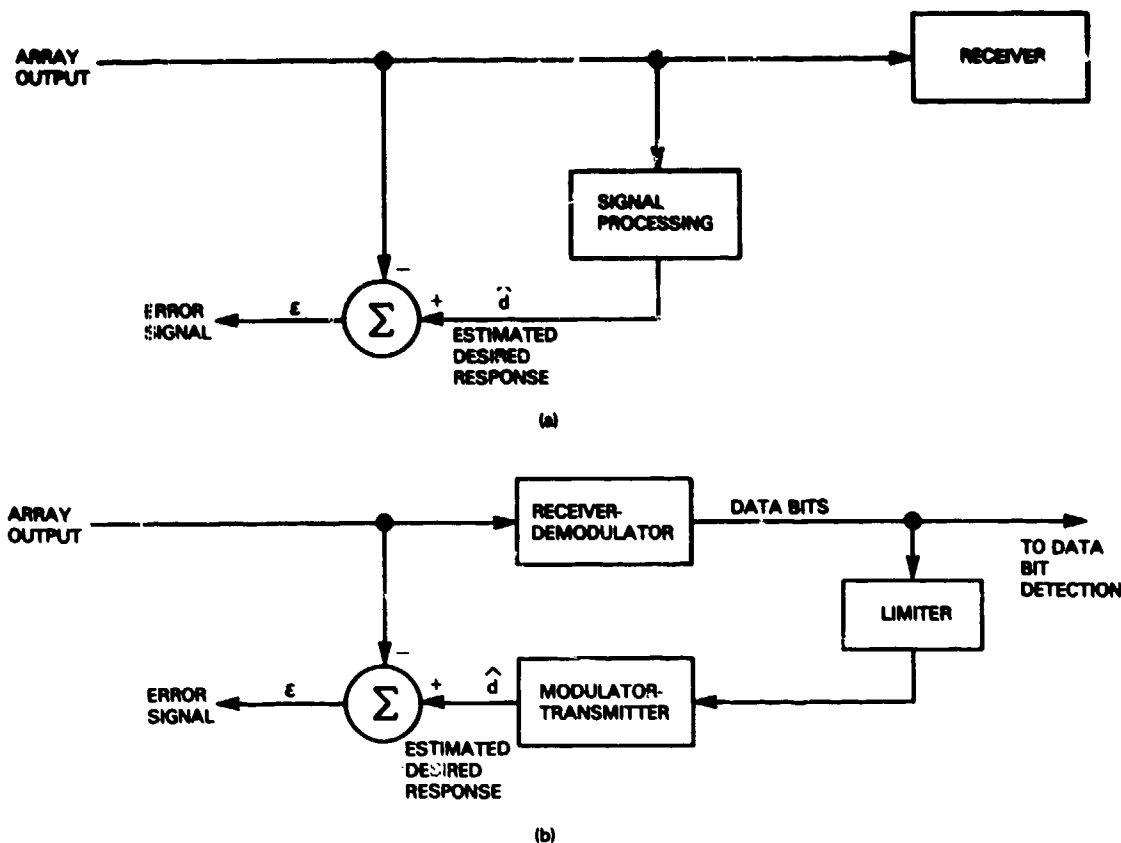
Figure 10. Generators of desired response.

due to the feedback generation of $\hat{d}$ should be considerably less than the data-modulation period. Details of the operation of an adaptive array in a spread-spectrum communication system and with feedback generation of the desired response are described by Compton.[10] For this system, the interference suppression due to the adaptive array is available in addition to the suppression afforded by the waveform processing.

4. ADAPTIVE NOISE CANCELLING

The adaptive element of a noise cancelling system[11] is a special case of figure 4(a). However, a noise cancelling system has such distinctive

[10] R. J. Compton, An Adaptive Array in a Spread-Spectrum Communication System, Proc. IEEE, 66 (March 1978), 289.

[11] B. Widrow et al, Adaptive Noise Cancelling: Principles and Applications, Proc. IEEE, 63 (December 1975), 1692.

features that it is treated separately in this section. In adaptive noise-cancelling systems, illustrated in figure 11, the desired response is replaced by the primary input, which is derived from a separate antenna. The primary input is excluded from weight vector calculations. Thus, the optimal weight vector for minimizing the mean square error is given by the Wiener-Hopf equation,

$$W_0 = R_{xx}^{-1} R_{xp} , \qquad (73)$$

where we interpret R_{xx} as the correlation matrix of the reference inputs, and R_{xp} is the correlation vector of the reference inputs with the primary input.

The system output for the adaptive noise-cancelling system is identical to the error signal. We show that minimizing the mean-square output is approximately equivalent to causing the output

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to be a minimum mean-square error estimate of the desired signal, d, if the adaptive filter output, y, is such that it has negligible correlation with d. We assume that the primary input is the sum of the desired signal and uncorrelated noise, n_o. Referring to figure 11, the canceller output is

$$\varepsilon = d + n_o - y . \qquad (74)$$

We assume that all signals are derived from stationary stochastic processes. Taking the expected value of the square of both sides of equation (74) gives

$$E[\varepsilon^2] = E[d^2] + E[(n_o - y)^2] +$$
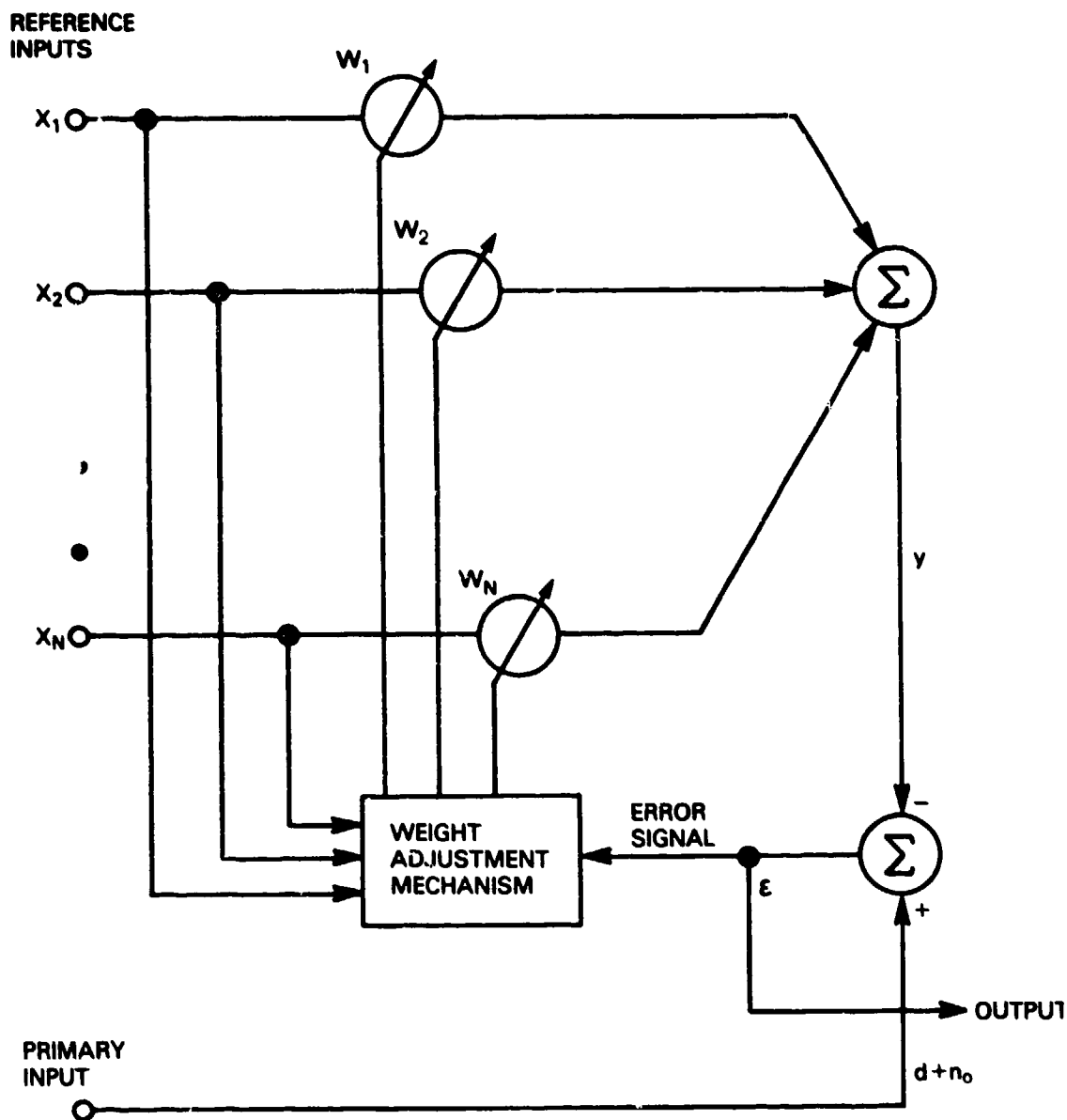$$2E[dn_o] - 2E[dy] . \qquad (75)$$

REFERENCE
INPUTS

Figure 11. Adaptive noise canceller.

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If d is uncorrelated with n_o and has negligible correlation with y, this equation reduces to

$$E[\varepsilon^2] = E[d^2] + E[(n_o - y)^2] \qquad (76)$$

The signal power, $E[d^2]$, is unaffected as the adaptive filter is adjusted to minimize $E[\varepsilon^2]$. Consequently, $E[(n_o - y)^2]$ is minimized when $E[\varepsilon^2]$ is minimized. From equation (74), it follows that $E[(\varepsilon - d)^2]$ is also minimized. We conclude that the adaptive noise canceller output is a minimum mean-square error estimate of the desired signal.

Since the Wiener-Hopf equation is valid, the Widrow-Hoff algorithm applies. Consider a continuous-time system with a two-dimensional reference vector, $X^T = [X_1, X_2]$, where $X_2(t)$ is the result of passing $X_1(t)$ through a quarter-wavelength delay. Equation (71) then implies the implementation of figure 1. Thus, we have shown how this configuration arises and why its performance can be expected to be nearly optimal. Using the Wiener-Hopf equation, Widrow[11] derived a Z-transform, sampled-data version of equation (34).

such as removing periodic interference from spread-spectrum communications. The adaptive notch filter has the form depicted in figure 12, which results from figure 11 when the reference inputs are derived from a tapped delay line that is fed by the delayed primary input. The delay Δ is sufficiently long to cause the desired wideband signal component of X_1 to become uncorrelated with the corresponding component of the primary input. However, because of their nature, periodic interference components in X_1 usually retain correlation with the corresponding components of the primary input. As a result, the tapped delay line adaptively forms a filter such that y is composed primarily of the periodic interference components. If the filter bandwidth is sufficiently narrow, the system output is the desired wideband signal with little distortion.

As shown in section 1, adaptive noise cancelling results in a small amount of cancellation of the desired signal. In terms of the overall antenna
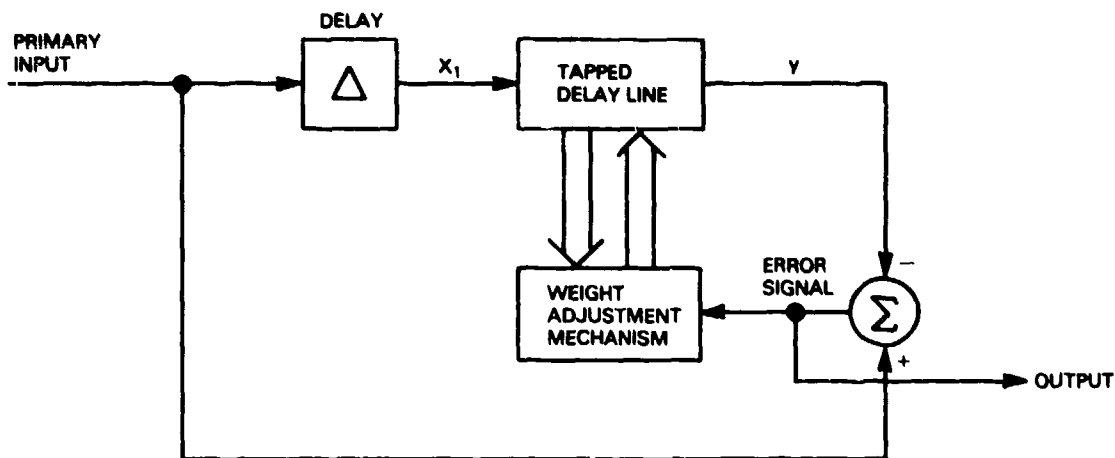

Figure 12. Adaptive notch filter.

The adaptive noise canceller can be used as an adaptive notch filter, which is useful in applications

[11] B. Widrow et al, Adaptive Noise Cancelling: Principles and Applications, Proc. IEEE, 63, (December 1975), 926.

pattern, this cancellation is caused by the changes in sensitivity of the main beam in the steering direction. These changes can occur because the mainlobe pattern is not constrained by the adaptive process.

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5. CONSTRAINED MINIMUM POWER CRITERION

A performance criterion that inherently limits the cancellation of the desired signal is the constrained minimum power criterion, which is applied to systems with adaptive elements of the form of figure 4(a). We assume that the components of X are derived from stationary stochastic processes. The cross-correlation matrix is defined by $R_{xx} = E[X^*X^T]$. The criterion requires the minimization of the mean output power,

$$E[|y|^2] = W' R_{xx} W , \qquad (77)$$

subject to the constraint

$$C'W = F , \qquad (78)$$

where F is a specified constraint vector and C is a matrix associated with the constraint. The specification of C and F depends upon the form of the adaptive system.

For the system of figure 6, equations (77) and (78) are applied to each summer output. Consider a specific summer and let W represent the associated complex weight vector and X the associated complex input vector. Assuming that the steering delays are such that the antenna array pattern points in the direction of the desired signal, the elements of X differ from each other only because of the interference and noise. Thus, if we require equation (78) with

$$C' = [1 \ 1 \ \dots \ 1], \ F = f , \qquad (79)$$

where f is a scalar, this constraint forces the desired signal component of the summer output to be a conventional coherent sum. Minimization of the output power then minimizes the interference power obtained from directions other than the main beam direction. Note that W has 2N degrees of freedom because of the N magnitudes and N phases of its elements. The complex constraint equation removes two of these degrees of freedom, but leaves $2N - 2$ to be used in limiting the interference.

For the system of figure 5, the constraint is somewhat different. We assume that identical desired signal components arriving on a plane wavefront appear at the first K taps simultaneously. Consequently, the filter response to the signal is equivalent to the response of a single tapped delay line in which each weight is equal to the sum of the weights in the corresponding vertical column of the adaptive element. These summation weights in the equivalent line are selected to give a desired frequency response. Thus, we require that

$$C_i' W = f_i , \ i = 1, 2, \dots , L , \qquad (80)$$

where f_i is a desired summation weight and C_i' is an N-dimensional vector of the form

$$C_i' = [0 \ 0 \ \dots \ 1 \ 1 \ \dots \ 1 \ 0 \ 0 \ \dots \ 0] \qquad (81)$$

The 1's correspond to the ith vertical column of K weights. To put equation (80) in matrix form, the constraint matrix is defined as

$$C = [C_1 \ C_2 \ \dots \ C_L] , \qquad (82)$$

which has the dimensions N by L. The L-dimensional vector of weights of the equivalent line is

$$F = \begin{bmatrix} f_1 \\ f_2 \\ . \\ . \\ . \\ f_L \end{bmatrix} \qquad (83)$$

With these definitions, the constraints can be combined into a single matrix equation of the form of equation (78). Once again, the constrained minimization ensures that the output power from all directions except boresight is minimized.

Having shown how the constraint equation is constructed, we proceed to a derivation of the

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optimal weight vector, assuming that the weights are fixed constants. For simpl- ~ity, we assume that all scalars and matrix components are real. By the method of Lagrange multipliers, we minimize

$$H = W^T R_{xx} W + \lambda^T (C^T W - F) \quad ,(84)$$

where λ is the vector of Lagrange multipliers.

Taking the gradient of equation (84) with respect to the vector W yields

$$\nabla H = 2 R_{xx} W + C \lambda . \quad (85)$$

A necessary condition for the minimum is determined by setting ∇H equal to zero. Thus,

$$W_\bullet = - \frac{1}{2} R_{xx}^{-1} C \lambda \quad . \quad (86)$$

Since $W_\bullet$ must satisfy the constraint, we substitute equation (86) into equation (78) with the result,

$$- \frac{1}{2} C^T R_{xx}^{-1} C \lambda = F . \quad (87)$$

This equation can be solved for λ. Substituting the solution into equation (86) gives

$$W_\bullet = R_{xx}^{-1} C (C^T R_{xx}^{-1} C)^{-1} F \quad , \quad (88)$$

where we assume that the indicated inverse exists. Frost's adaptive algorithm,[12] which is based upon approximating equation (88), is

$$W(0) = B \quad ,$$

$$W(k + 1) =$$

$$A[W(k) - \mu y(k) X(k)] + B \quad , \quad (89)$$

where

$$A = I - C(C^T C)^{-1} C^T \quad ,$$

$$B = C(C^T C)^{-1} F . \quad (90)$$

The algorithm ensures the satisfaction of the constraint after each iteration and has other desirable properties. Neither pilot signals nor generators of the desired response are needed in constrained-minimum-power systems.

In the development of the adaptive algorithms, it has always been assumed that the noise processes are stationary. However, the main purpose of adaptive systems is to automatically adjust to nonstationary inputs, especially when the stochastic properties are unknown *a priori*. Fortunately, experimental results and tentative theoretical results seem to indicate that adaptive algorithms retain their usefulness in many realistic environments with nonstationary inputs.[13]

In addition to the stochastic properties of the inputs, we have tacitly assumed plane-wave signals, an ideal propagation medium, and distortionless receivers. Since these assumptions may not be valid under actual operating conditions, Vural has investigated the effects of signal, medium, and system deviations from the usual assumptions.[14]

Adaptive antenna systems have many applications. In general, adaptation is potentially helpful for communications or radar when the desired signal and the interferences are distinguishable *a priori*. The discriminant may be the direction of arrival, a waveform characteristic, or even signal polarization.[1]

[1] B. D. Steinberg, *Principles of Aperture and Array System Design*, Wiley, 1976.

[13] B. Widrow et al. Stationary and Nonstationary Learning Characteristics of the LMS Adaptive Filter, Proc. IEEE, 63 (August 1976), 1151.

[14] A. M. Vural, Effects of Perturbations on the Performance of Optimum Adaptive Arrays, IEEE Trans. Aerospace Electron. Syst., AES-15 (January 1979), 76.

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13. B. Widrow et al, *Stationary and Nonstationary Learning Characteristics of the LMS Adaptive Filter*, Proc. IEEE, 64 (August 1976), 1151.

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